LINEAR ALGEBRA WORKSHEET 5

MATH1014 SPRING SESSION

(1) Find all eigenvalues of the matrix. Is the matrix diagonalisable? If so, find an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$.

$$A = \begin{bmatrix} 0 & -1 & -1 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}.$$

(2) (a) Suppose that A is diagonalisable and has only one eigenvalue λ. Explain why $A = \lambda I$.

(b) Is the matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

diagonalisable?

(3) Which of the following are possible for a 3×3 matrix? If it's impossible, explain why. If it's possible, give an example.

(a) A has exactly two distinct eigenvalues and is diagonalisable

(b) B has exactly four distinct eigenvalues and is diagonalisable

(c) C has exactly three distinct eigenvalues and is not diagonalisable

(d) D has exactly two distinct eigenvalues and is not invertible.

(4) For which values of t does the matrix

$$\begin{bmatrix} 1 & t \\ 2 & 3 \end{bmatrix}$$

have two distinct eigenvalues? (Hint: a quadratic equation $ax^2 + bx + c = 0$ has two different real solutions exactly when $b^2 > 4ac$).