LINEAR ALGEBRA WORKSHEET 6

MATH1014 SPRING SESSION

(1) Suppose $T : V \to W$ is a linear transformation, $\mathscr{B} = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$ is a basis for V, and $\mathscr{C} = \{\mathbf{c}_1, \mathbf{c}_2\}$ is a basis for W. If the matrix of T with respect to $\mathscr{B}$ and $\mathscr{C}$ is

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 5 \end{bmatrix}$$

then find $T(\mathbf{b}_1 - \mathbf{b}_3)$ in terms of $\mathbf{c}_1$ and $\mathbf{c}_2$.

(2) Consider the linear transformation $T : \mathbb{P}_2 \to \mathbb{P}_2$ given by $T(p(t)) = p(t) + tp'(t) + p''(t)$. Fix the standard basis $\mathscr{B} = \{1, t, t^2\}$ for $\mathbb{P}_2$.

(a) Find $[T]_{\mathscr{B}}$.

(b) Does there exist a non-zero polynomial q in $\mathbb{P}_2$ such that $T(q) = 3q$? If so, find one.

(3) The matrix $B = \begin{bmatrix} 0.4 & -0.3 \\ 0.4 & 1.2 \end{bmatrix}$ has eigenvectors $\mathbf{u} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$. Use diagonalisation to explain why B^k approaches $\begin{bmatrix} -0.5 & -0.75 \\ 1.0 & 1.5 \end{bmatrix}$ as $k \to \infty$.

(4) Find the eigenvalues and eigenspaces of the matrix $A = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$.