Math 110, Fall 2012, Sections 109-110
Worksheet 1

1. Let V be a vector space over a field F. Carefully prove the following statement using only the axioms of vectors spaces and fields: Suppose $a \in F$ and $b \in V$. Then $ab = 0$ if and only if $a = 0$ or $b = 0$.

2. Why is the following question ill-posed (i.e. why doesn't it make any sense): "Let V be the set of cows in California. Is V a vector space over $\mathbb{F}_7$?"

3. Are the following vector spaces over $\mathbb{R}$?

   (a) The set of all real 2×2 matrices of the form
   $$\begin{pmatrix} a & 2 \\ 2 & b \end{pmatrix}$$
   with the usual matrix addition and scalar multiplication.

   (b) The set of all real 2×2 matrices of the form
   $$\begin{pmatrix} a & a+b \\ a+b & b \end{pmatrix}$$
   with the usual matrix addition and scalar multiplication.

   (c) The set of all real 2×2 matrices of the form
   $$\begin{pmatrix} a^2 & a \\ b & 0 \end{pmatrix}$$
   with the usual matrix addition and scalar multiplication.

4. Consider the subspaces of $M_{n\times n}(F)$ consisting: W_0, consisting of traceless matrices, W_1 consisting of matrices (a_{ij}) with $a_{ii} = 0$ for all i, and W_2 consisting of all strictly upper triangular matrices. Prove that $W_2 \subseteq W_0 \cap W_1$.

5. Let $W_1 = \{(\alpha, \alpha) : \alpha \in \mathbb{R}\}$ and let $W_2 = \{(\beta, -\beta) : \beta \in \mathbb{R}\}$. Prove that $\mathbb{R}^2 = W_1 \oplus W_2$.