Math 110, Fall 2012, Sections 109-110
Worksheet 12

1. True or false? Assume V is a finite-dimensional inner product space over $\mathbb{C}$ and that T is a linear operator on V.
   (a) If T is self-adjoint and β is a basis for V, then $[T]_\beta$ is self-adjoint.
   (b) If x is an eigenvector for T, then x is also an eigenvector for T^*.
   (c) If T and S are self-adjoint, then $T + S$ is self-adjoint.
   (d) If T is self-adjoint and $c \in F$, then cT is self-adjoint
   (e) If T and S are normal, then $T + S$ is normal.
   (f) If T is normal and $c \in F$, then cT is normal.
2. Let T be a normal operator on a complex finite-dimensional inner product space V. Prove that T is self-adjoint if and only if all the eigenvalues of T are real numbers.
3. Suppose that $A \in M_{n\times n}(\mathbb{C})$. Prove that $A^*A = I_n$ if and only if the columns of A form an orthonormal basis for $\mathbb{C}^n$.
4. Let T be a self-adjoint linear operator on an inner product space V. Suppose W is a T-invariant subspace of V. Prove that $W^\perp$ is also T-invariant.