Math 110, Fall 2012, Sections 109-110
Worksheet 2

1. Prove that if $\{A_1, A_2, \ldots, A_k\}$ is a linearly independent subset of $M_{n\times n}(F)$ then so is $\{A_1^t, \ldots, A_k^t\}$.

2. Determine if the statement is true or false, and justify your answer. Let S and T be subsets of a vector space V.

   (a) If S is linearly independent and $S \subset T$, then T is linearly independent.

   (b) If S is linearly dependent and $S \subset T$, then T is linearly dependent.

   (c) If S spans V and $S \subset T$, then T spans V.

   (d) If S does not span V and $S \subset T$, then T does not span V.

3. Let V be a vector space. Assume $u, v, w \in V$ and $u + v + w = 0$. Let $W_1 = \operatorname{span}\{u, v\}$ and $W_2 = \operatorname{span}\{u, w\}$. Is $W_1 = W_2$? Prove your answer.

4. (a) As a subset of the vector space $\mathbb{R}^3$, is $\{(2, 4, 1), (2, 3, 4)\}$ linearly independent or linearly dependent?

   (b) Compute $4(2, 4, 1) - (1, 2, 4)$ in $\mathbb{Z}/7$ and comment.

5. Let V be the real vector space of functions from $\mathbb{R}$ to $\mathbb{R}$ with pointwise addition and scalar multiplication. Let $f, g, h \in V$ be the elements $f(x) = e^x$, $g(x) = \cos(x)$, and $h(x) = \sin(x)$. Is $f \in \operatorname{span}\{g, h\}$?