Math 110, Fall 2012, Sections 109-110
Worksheet 6

1. Let $A = \begin{pmatrix} 1 & 4 \\ 1 & 2 \end{pmatrix}$.
   (a) Write A^{-1} as a product of elementary matrices.
   (b) Write A as a product of elementary matrices

2. True or false? If true, provide proof. If false, provide a counterexample.
   (a) If A is an $m \times n$ matrix with a set of three linearly independent columns, then it also has a set of three linearly independent rows.
   (b) Elementary row operations preserve the rank of A.
   (c) Elementary column operations preserve the rank of A.
   (d) Elementary row operations preserve the range of A.
   (e) Elementary column operations preserve the range of A.
   (f) Every $n \times n$ matrix can be written as a product of elementary matrices.

3. For each of the following linear transformations, determine if T is invertible and compute T^{-1} if applicable.
   (a) $T : P_3(\mathbb{R}) \to P_3(\mathbb{R})$ defined by $T(p) = p'$.
   (b) $T : P_2(\mathbb{R}) \to \mathbb{R}^3$ defined by $T(p) = (p(-1), p(0), p(1))$.

4. Suppose that D can be transformed into B using row and column operations. Prove that D' can be transformed into B' using row and column operations, where

$$D' = \left(\begin{array}{c|c} 1 & 0 \\ \hline 0 & D \end{array}\right), \quad B' = \left(\begin{array}{c|c} 1 & 0 \\ \hline 0 & B \end{array}\right).$$

5. Suppose that $A \in M_{m\times n}(F)$ and $b \in F^m$. Prove that if $\operatorname{rank}(A \mid b) = \operatorname{rank} A$, then there exists $x \in F^n$ such that $Ax = b$.