Math 110, Fall 2012, Sections 109-110
Worksheet 7

1. What does it mean for two systems of equations to be *equivalent*? Give an example of two distinct but equivalent systems of linear equations.

2. (a) How do you find a basis for the column space of a matrix? Carefully justify why your method works, citing theorems where appropriate.

   (b) How do you find a basis for the null space of a matrix? Carefully justify why your method works, citing theorems where appropriate.

   (c) Apply your methods to find bases for the column space and the null space of

$$A = \begin{pmatrix} 1 & 2 & 3 & 0 \\ -2 & 4 & 10 & 8 \\ 1 & -2 & -5 & -4 \end{pmatrix}.$$

3. Are the following statements true or false? If true, justify your answer. If false, provide a counterexample.

   (a) If A is row equivalent to A', then $Ax = b$ is consistent if and only if $A'x = b$ is consistent.

   (b) The $n \times n$ matrix A is invertible if $Ax = 0$ has the trivial solution.

4. Prove that $Ax = b$ is consistent if and only if $\operatorname{rank} A = \operatorname{rank}(A \mid b)$.

5. Suppose $A \in M_{n\times n}(\mathbb{R})$ and $b \in \mathbb{R}^n$. Prove that if $Ax = b$ is consistent, then it either has one solution, or infinitely many solutions. For bonus points, use the words“homogeneous” in your response.