Math 110, Fall 2012, Sections 109-110
Worksheet 8

1. True or false? If true, justify. If false, provide a counterexample.

   (a) The determinant of an elementary matrix can be any element of the field.

   (b) A matrix $A \in M_{n \times n}(F)$ has rank n if and only if $\det A \neq 0$.

   (c) The determinant $\det : M_{n \times n}(F) \to F$ is a linear functional.

   (d) If $c \in F$, then $\det(cA) = c \det(A)$.

   (e) If you interchange two columns of a matrix, then the resulting matrix has determinant that is the opposite of the determinant of the original matrix.

2. A matrix $M \in M_{n \times n}(F)$ is called *nilpotent* if there exists a $k > 0$ for which $M^k = 0$. What can you say about the determinant of a nilpotent matrix?

3. Suppose $A, B \in M_{2013 \times 2013}(\mathbb{R})$. Prove that we cannot have $AB = -BA$. Bonus: construct two by two matrices A and B with $AB = -BA$.

4. Suppose that $D \in M_{n \times n}(F)$ is upper-triangular. In terms of the entries of D, what is the determinant of D?

5. Suppose C is an $m \times m$ matrix. Calculate the determinant of the $(n + m)$ by $(n + m)$ matrix
$$C' = \begin{pmatrix} C & B \\ 0 & I_n \end{pmatrix}.$$