Math 1B, Fall 2008
Section 107

Quiz 10 Solutions

(1) *Find*

$$\int \frac{y}{y^2 - y}\, dy.$$

No tricks here...

$$\int \frac{y}{y^2 - y}\, dy = \int \frac{dy}{y - 1} = \ln|y - 1| + C.$$

(2) *Solve as an implicit function of y and θ:*

$$\frac{dy}{d\theta} = \frac{e^y \sin^2\theta}{y \sec\theta}.$$

Separating variables gives

$$\frac{y\, dy}{e^y} = \frac{\sin^2\theta\, d\theta}{\sec\theta}.$$

Now integrate each side separately

$$\begin{aligned}
\int \frac{y\, dy}{e^y} &= \int ye^{-y}\, dy \\
&= -ye^{-y} - \int -e^{-y}\, dy \\
&= -ye^{-y} - e^{-y} \\
&= -(y+1)e^{-y}.
\end{aligned}$$

and also (using the subtitution $u = \sin\theta$)

$$\begin{aligned}
\int \frac{\sin^2\theta\, d\theta}{\sec\theta} &= \int \sin^2\theta\cos\theta\, dy \\
&= \int u^2\, du \\
&= \frac{1}{3}\sin^3\theta.
\end{aligned}$$

So our final answer is

$$-(y+1)e^{-y} = \frac{1}{3}\sin^3\theta + C$$

(3) *A force of 10 lb is needed to stretch a spring 4 inches from its rest length. How much work is done in stretching it 6 inches from rest length?*

When holding a spring stretched, the required force is $F = kx$, so the first fact tells us 10 lb $= k(4$ in), or $k = \frac{5}{2}$ lb/in. So the work to stretch 6 inches from rest length is

$$W = \int_0^6 \frac{5}{2} x \, dx,$$

where x is the amount of extension from rest. Evaluating the integral, we get that $W = 45$ in·lb. Alternatively, we could convert to feet and get that $k = 30$ lb/ft, and

$$W = \int_0^{\frac{1}{2}} 30x \, dx,$$

which yields $W = 15/4$ ft·lb.