Math 1B, Fall 2008
Section 108

Quiz 1 Solutions

(1) *Find* $\int xe^{-x}dx$.

Use integration by parts:

$$\int xe^{-x}dx = -xe^{-x} - \int -e^{-x}dx \qquad \left|\begin{array}{ll} u = x & dv = e^{-x}dx \\ du = dx & v = -e^{-x} \end{array}\right|$$

$$= -xe^{-x} - e^{-x} + C$$

$$= -(x+1)e^{-x} + C$$

(2) *Evaluate*

$$\int_0^{\ln\sqrt{3}} \frac{e^x}{1+e^{2x}}\, dx.$$

Recall that $e^{2x} = (e^x)^2$, and integrate using a u-substitution:

$$\left|\begin{array}{ll} u = e^x & u(0) = e^0 = 1 \\ du = e^x\, dx & u(\ln\sqrt{3}) = e^{\ln\sqrt{3}} = \sqrt{3} \end{array}\right|$$

$$\begin{aligned} \int_0^{\ln\sqrt{3}} \frac{e^x}{1+(e^x)^2}\, dx &= \int_1^{\sqrt{3}} \frac{du}{1+u^2}\, du \\ &= \tan^{-1} u\Big|_1^{\sqrt{3}} \\ &= \tan^{-1}\sqrt{3} - \tan^{-1} 1 \\ &= \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12} \end{aligned}$$

(3) *Find* $\int \sin\sqrt{x}\ dx$.

First a substitution:

$$\int \sin\sqrt{x}\ dx = 2\int r\sin r\ dr \qquad \left|\begin{array}{c} r = \sqrt{x} \\ dr = \dfrac{dx}{2\sqrt{x}} = \dfrac{dx}{2r} \\ dx = 2r\ dr \end{array}\right|$$

Now integrate by parts:

$$\begin{aligned} 2\int r\sin r\ dr &= 2\left(-r\cos r - \int -\cos r dr\right) \\ &= 2\left(-r\cos r + \int \cos r dr\right) \\ &= 2\left(-r\cos r + \sin r\right) + C \\ &= 2\left(\sin\sqrt{x} - \sqrt{x}\cos\sqrt{x}\right) + C \end{aligned} \qquad \left|\begin{array}{cc} u = r & dv = \sin r dr \\ du = dr & v = -\cos r \end{array}\right|$$