Math 1B, Fall 2008
Section 107

Quiz 2 Solutions

(1) *Find* $\int \tan^3 x \sec x \; dx$.

Using the fact that $\tan^2 x = \sec^2 x - 1$ and the subtitution $\begin{bmatrix} u = \sec x \\ du = \sec x \tan x \; dx \end{bmatrix}$ we get

$$\begin{aligned} \int \tan^3 x \sec x \; dx &= \int (\sec^2 x - 1) \tan x \sec x \; dx \\ &= \int u^2 - 1 \; du \\ &= \frac{1}{3}u^3 - u + C \\ &= \frac{1}{3}\sec^3 x - \sec x + C \end{aligned}$$

(2) *Find* $\int \frac{\tan(1/z)}{z^2} \; dz$.

Using the subtitution $\begin{bmatrix} u = \frac{1}{z} \\ du = -\frac{1}{z^2}dz \end{bmatrix}$ we get

$$\begin{aligned} \int \frac{\tan(1/z)}{z^2} \; dz &= -\int \tan u \; du \\ &= -\ln|\sec u| + C \\ &= -\ln\left|\sec\frac{1}{z}\right| + C \\ &= \ln\left|\cos\frac{1}{z}\right| + C \end{aligned}$$

Remarks: The integral $\int \tan u \; du$ is on the cheat sheet. Both $-\ln\left|\sec\frac{1}{z}\right|$ and $\ln\left|\cos\frac{1}{z}\right|$ are acceptable answers.

(3) *Find* $\int \frac{1}{\sqrt{t^2 - 6t + 13}}\, dt$.

First complete the square: $t^2 - 6t + 13 = (t-3)^2 + 4$. Now substitute $\begin{bmatrix} u = t - 3 \\ du = dt \end{bmatrix}$ to get

$$\begin{aligned} \int \frac{1}{\sqrt{t^2 - 6t + 13}}\, dt &= \int \frac{1}{\sqrt{(t-3)^2 + 4}}\, dt \\ &= \int \frac{1}{\sqrt{u^2 + 4}}\, du. \end{aligned}$$

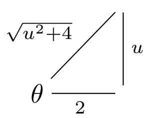
This is now a trig substitution problem. We will make the substitution

$$\begin{bmatrix} u = 2\tan\theta & \quad -\pi/2 < \theta < \pi/2 \\ du = 2\sec^2\theta\ d\theta & \end{bmatrix},$$

and use the fact that $\sqrt{1 + \tan^2\theta} = \sqrt{\sec^2\theta}$ which is equal to $\sec\theta$ on the chosen domain.

$$\begin{aligned} \int \frac{1}{\sqrt{u^2 + 4}}\, du &= \int \frac{2\sec^2\theta}{\sqrt{4\tan^2\theta + 4}}\, d\theta \\ &= \int \frac{2\sec^2\theta}{2\sqrt{\tan^2\theta + 1}}\, d\theta \\ &= \int \frac{\sec^2\theta}{\sec\theta}\, d\theta \\ &= \int \sec\theta\ d\theta \\ &= \ln\left|\sec\theta + \tan\theta\right|. \end{aligned}$$

Reversing our substitution, $\tan\theta = u/2 = (t-3)/2$. To find $\sec\theta$, we use the triangle




we can read off $\sec\theta = \dfrac{\sqrt{u^2+4}}{2} = \dfrac{\sqrt{(t-3)^2+4}}{2}$. So putting it all together we get

$$\int \frac{1}{\sqrt{t^2 - 6t + 13}}\, dt = \ln\left|\frac{\sqrt{(t-3)^2+4}}{2} + \frac{t-3}{2}\right| + C$$

This can optionally be simplified to

$$\ln\left|\sqrt{(t-3)^2+4} + t - 3\right| + C_2.$$

(Why?)