Math 1B, Fall 2008
Section 107

Quiz 3 Solutions

(1) *Find* $\int_{-\infty}^{\infty} xe^{-x^2}\ dx$.

First find the antiderivitive (via the substitution $u = -x^2$)

$$\int xe^{-x^2}\ dx = -\frac{1}{2}e^{-x^2}.$$

Now we split the integral and take the two limits seperately. Using different variable names here is optional, but I think it helps with clarity:

$$\begin{aligned}
\int_{-\infty}^{\infty} xe^{-x^2}\ dx &= \int_{-\infty}^{0} xe^{-x^2}\ dx + \int_{0}^{\infty} xe^{-x^2}\ dx \\
&= \lim_{a\to\infty}\int_{-a}^{0} xe^{-x^2}\ dx + \lim_{b\to\infty}\int_{0}^{b} xe^{-x^2}\ dx \\
&= \lim_{a\to\infty} -\frac{1}{2}e^{-x^2}\Big]_{-a}^{0} + \lim_{b\to\infty} -\frac{1}{2}e^{-x^2}\Big]_{0}^{b} \\
&= \lim_{a\to\infty}\left(\frac{1}{2}e^{-a^2} - \frac{1}{2}\right) + \lim_{b\to\infty}\left(\frac{1}{2} - \frac{1}{2}e^{-b^2}\right) \\
&= 0 - \frac{1}{2} + \frac{1}{2} - 0 = 0
\end{aligned}$$

Bonus: find a quick way to get this answer using the fact that xe^{-x^2} is an *odd function* (that is, $f(-x) = -f(x)$).

(2) *Find* $\int \frac{r^2}{r+4}\ dr$.

Solution 1: Use polynomial long devision to find

$$\frac{r^2}{r+4} = r - 4 + \frac{16}{r+4}.$$

Thus

$$\begin{aligned}
\int \frac{r^2}{r+4}\ dr &= \int r - 4 + \frac{16}{r+4}\ dr \\
&= \frac{1}{2}r^2 - 4r + 16\ln|r+4| + C
\end{aligned}$$

Solution 2: Substitute $u = r + 4$ (which gives $du = dr$ and $u = r - 4$). Now we can calculate

$$\begin{aligned}
\int \frac{r^2}{r+4}\, dr &= \int \frac{(u-4)^2}{u}\, du \\
&= \int \frac{u^2 - 8u + 16}{u}\, du \\
&= \frac{1}{2}u^2 - 8u + 16 \ln |u| + C \\
&= \frac{1}{2}(r+4)^2 - 8(r+4) + 16 \ln |r+4| + C.
\end{aligned}$$

This answer doesn't look the same as solution 1, but if you expand it out, they only differ by a constant.

(3) *How large must we choose n so that $|E_L| \leq \frac{1}{7}$ in evaluating $\int_0^1 x^2\, dx$?*

We know $|E_L| \leq \dfrac{K_1(b-a)^2}{2n}$, and we want $|E_L| \leq \dfrac{1}{7}$, so we find n big enough that $\dfrac{K_1(b-a)^2}{2n} \leq \dfrac{1}{7}$. In this case, $b - a = 1$, so all we need to find is K_1:

$$K_1 = \max_{0 \leq x \leq 1} |f'(x)| = \max_{0 \leq x \leq 1} |2x| = 2$$

So

$$\frac{K_1(b-a)^2}{2n} = \frac{1}{n}.$$

To find when $\dfrac{1}{n} \leq \dfrac{1}{7}$, multiply both sides by $7n$ to get $7 \leq n$. Thus $n = 7$ is the smallest acceptable choice. (Notice that when we took the reciprocal, the direction of the inequality changed.)