Math 1B, Fall 2008
Section 107

Quiz 6 Solutions

(1) *Determine whether the following series converges or diverges,*

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+1}}.$$

Thinking that $\sqrt{n^2+1} \approx \sqrt{n^2} = n$ for large n, we apply the Limit Comparison Test with

$$a_n = \frac{1}{\sqrt{n^2+1}} \qquad b_n = \frac{1}{n}.$$

The test applies, as $a_n, b_n \geq 0$ and

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{n}{\sqrt{n^2+1}} = \lim_{n\to\infty} \frac{1}{\sqrt{1+n^{-2}}} = 1.$$

Since $\sum_{n=1}^{\infty} b_n$ diverges (it is the Harmonic Series/by p-test), $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+1}}$ also diverges by the Limit Comparison Test.

(2) *Determine whether the following series is absolutely convergent, conditionally convergent, or divergent,*

$$\sum_{n=1}^{\infty} \frac{5^{n-1}}{n^2 4^n}.$$

We use the ratio test. If $a_n = \frac{5^{n-1}}{n^2 4^n}$,then

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \frac{5^n}{(n+1)^2 4^{n+1}} \cdot \frac{4^n n^2}{5^{n-1}} = \lim_{n\to\infty} \frac{5}{4} \frac{n^2+2n+1}{n^2} = \frac{5}{4}.$$

Thus by the ratio test, $\sum_{n=1}^{\infty} \frac{5^{n-1}}{n^2 4^n}$ diverges.

(3) *Determine whether the following series is absolutely convergent, conditionally convergent, or divergent*

$$\sum_{n=1}^{\infty} \frac{(-1)^n \arctan n}{n^2}.$$

If $a_n = \frac{(-1)^n \arctan n}{n^2}$, then

$$|a_n| \leq \frac{\pi}{2n^2}$$

because $|\arctan x| \leq \frac{\pi}{2}$ for all real numbers x. By the p-test,

$$\sum_{n=1}^{\infty} \frac{\pi}{2n^2} = \frac{\pi}{2} \sum_{n=1}^{\infty} \frac{1}{n^2}$$

converges. Thus $\sum_{n=1}^{\infty} |a_n|$ converges by the Comparison Test, and we conclude that

$$\sum_{n=1}^{\infty} \frac{(-1)^n \arctan n}{n^2}$$

is absolutely convergent.

This problem could also be done by limit comparison.