Math 1B, Fall 2008
Sections 107 and 108
In-class exercises from Sept 8, 2008

(1) Find

$$\int x^3\sqrt{x^2+4}dx.$$

Substitute $x = 2\tan\theta$ with $-\pi/2 < \theta < \pi/2$. This gives $dx = 2\sec^2\theta d\theta$. Then

$$\begin{aligned}
\int x^3\sqrt{x^2+4}dx &= \int (2\tan\theta)^3\sqrt{4\tan^2\theta+4}(2\sec^2\theta)d\theta \\
&= 32\int \tan^3\theta\sec^2\theta\sqrt{\sec^2\theta}d\theta \\
&= 32\int \tan^3\theta\sec^3\theta d\theta \\
&= 32\int (\sec^2\theta-1)\sec^3\theta\tan\theta.
\end{aligned}$$

where we used that $\sec\theta > 0$ in the domain of θ. Now substitute $u = \sec\theta$ to get

$$\begin{aligned}
32\int (\sec^2\theta-1)\sec^3\theta\tan\theta &= 32\int u^4 - u^2du \\
&= 32/5u^5 - 32/3u^3 \\
&= 32/5\sec^5\theta - 32/3\sec^3\theta.
\end{aligned}$$

Using the triangle with legs x and 2 and hypotenuse $\sqrt{x^2+4}$, we get $\sec\theta = (1/2)\sqrt{x^2+4}$. Plugging in gives the answer

$$\frac{1}{5}(x^2+4)^{5/2} - \frac{4}{3}(x^2+4)^{3/2} + C.$$

(2) Find $\int \frac{1}{t\sqrt{t^2-1}}dt$.

Making the subtitution $t = \sec\theta$ with $0 \le \theta < \pi/2$ or $\pi \le \theta < 3\pi/2$ gives

$$\begin{aligned}
\int \frac{1}{t\sqrt{t^2-1}}dt &= \int \frac{\sec\theta\tan\theta d\theta}{\sec\theta\sqrt{\sec^2\theta - 1}} \\
&= \int \frac{\tan\theta d\theta}{\sqrt{\tan^2\theta}} \\
&= \int \frac{\tan\theta d\theta}{\tan\theta} \\
&= \int d\theta \\
&= \theta \\
&= \sec^{-1} t + C.
\end{aligned}$$