Name: ______________________________

Math 32, Spring 2010, Section 101
Quiz 4 Solution

(1) (2 pts) Sketch a graph of the curve $y = (x-2)^2 + 1$. Be sure to include the coordinates of any x-intercept(s), y-intercept(s), and the vertex.

The graph is $y = x^2$, translated 2 to the right and 1 up. From the graph, we can see that there are no x-intercepts. The y-intercept is found by plugging in $x = 0$, which gives $(0, 5)$. The equation $(x-2)^2 + 1$ is in "verex form" already, and we can read off the vertex $(2, 1)$.

(2) (3 pts) A piece of wire 16in. long is to be cut into two pieces. Let x denote the length of the first piece, and $16 - x$ denote the length of the second. The first piece is to be bent into a circle, and the second piece into a square. Express the total combined area of the circle and square as a function of x. (For this quiz, don't worry too much about simplifying your expression for area, as long as it is in terms of just x.)

The circle has perimeter x, and let's call its radius r. We then have $x = 2\pi r$. The square has perimeter $16 - x$, and let's call its side legnth s. We then have $16 - x = 4s$. The combined area is $A = \pi r^2 + s^2$. Solving the constraints for r and x we get

$$A = \pi \left(\frac{x}{2\pi}\right)^2 + \left(\frac{16-x}{4}\right)^2 = \frac{x^2}{4\pi} + \frac{(16-x)^2}{16}.$$

(3) (5 pts) Which point on the curve $y = \sqrt{x-2} + 1$ is closest to the point $(4, 1)$?

Our target equation, the distance between (x, y) and $(4, 1)$, is given by

$$d = \sqrt{(x-4)^2 + (y-1)^2}.$$

Our constraint is that $y = \sqrt{x-2} + 1$. Plugging this in, we get

$$d = \sqrt{(x-4)^2 + (\sqrt{x-2} + 1 - 1)^2} = \sqrt{(x-4)^2 + x - 2} = \sqrt{x^2 - 7x + 14}.$$

To minimize the square root, it's enough to minimize the thing inside the square root. So the x-value of the vertex of the inner quadratic is $7/2$. To find the y-coordinate, we plug back into the original equation to get $y = \sqrt{7/2 - 2} + 1 = \sqrt{3/2} + 1$. Our answer is then $\left(\frac{7}{2}, \sqrt{\frac{3}{2}} + 1\right)$.