Name: ______________________________

Math 32, Spring 2010, Section 101
Quiz 6

(1) (3 pts) Find the domains of the following functions.

a) $f(x) = \log_{10}(3 - 4x)$ b) $g(x) = \ln(x^2)$ c) $h(x) = \log_3(e^x + 1)$

a) This function is defined when $3 - 4x > 0$. That is, when $x < 3/4$. In interval notation, that's $(-\infty, 3/4)$.

b) This function is defined when $x^2 > 0$. That is, when $x \neq 0$. In interval notation, that's $(-\infty, 0) \cup (0, \infty)$.

c) This is defined when $e^x + 1 > 0$. This is always true, so the domain is all real numbers. In interval notation, $(-\infty, \infty)$.

(2) (3 pts) Simplify the expression as much as possible by using the definition and properties of logarithms.

a) $\log_{10} 70 - \log_{10} 7$ b) $2^{\log_2 5} - 3\log_5 \sqrt[3]{5}$ c) $\log_2(8) \cdot \ln(\frac{1}{e})$

a) $\log_{10} 70 - \log_{10} 7 = \log_{10}(70/7) = \log_{10} 10 = 1$

b) $2^{\log_2 5} - 3\log_5 \sqrt[3]{5} = 5 - \log_5 \sqrt[3]{5}^3 = 5 - \log_5 5 = 5 - 1 = 4$

c) $\log_2(8) \cdot \ln(\frac{1}{e}) = 3 \cdot \ln(e^{-1}) = 3 \cdot (-\ln(e)) = 3 \cdot (-1) = -3.$

(3) (4 pts) Solve the inequality $\log_2 \dfrac{2x-1}{x-2} < 0$. For full credit, be sure to check the domain.

Exponentiating both sides, we get $\dfrac{2x-1}{x-2} < 1$, or $\dfrac{2x-1}{x-2} - 1 < 0$. Putting over a common denominator, we get

$$\frac{2x-1}{x-2} - 1 = \frac{2x-1}{x-2} - \frac{x-2}{x-2} = \frac{x+1}{x-2} < 0.$$

Using the method of key numbers, we get that this is true when $-1 < x < 2$.

Now we need to check that these solutions make sense in the original inequality by finding the domain of the left-hand side. The expression is defined when $\dfrac{2x-1}{x-2} > 0$. Using the method of key numbers, this holds when $x > 2$ and when $x < \frac{1}{2}$. So the only time our solution above is defined is when $-1 < x < \frac{1}{2}$, or on the interval $(-1, 1/2)$.