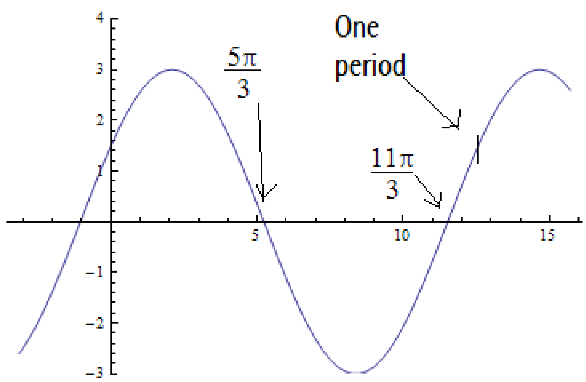
Math 32, Spring 2010, Section 101
Worksheet 11 Solutions

Work through the following problems in groups of about four. Take turns writing; everyone should get a chance to write for some of the problems. It's more important to understand the problems than to do all of them.

1. Graph $y = 3\sin(\frac{1}{2}x + \pi/6)$. Indicate the period, amplitude, and label the x-intercepts for one period. Show me your graph before moving on to #2.



The period is $2\pi/(1/2) = 4\pi$ and the amplitude is 3. The first peak occurs at $x = 2\pi/3$ and the first valley at $x = 8\pi/3$.

2. Evaluate the following if they are defined. If they are undefined, say why.

(a) $\cos^{-1}(-1)$

$\cos^{-1}(-1)$ is the value of x such that $0 \le x \le \pi$ and $\cos x = -1$. In this case, it's π because $\cos \pi = -1$.

(b) $\cos(\arccos(\pi))$

This is undefined, since the domain of arccos is $[0, \pi]$. In other words, there is no number x such that $\cos(x) = \pi$.

(c) $\tan(\tan^{-1}(4))$

This is equal to 4. The domain of $\tan^{-1}$ is all real numbers, so no problems there. $\tan^{-1}$ is defined so that when you apply tan to $\tan^{-1}(x)$, you get x back. However, this doesn't work in the opposite order (applying arctan to $\tan(x)$ - see the next problem for an example of this involving cosine).

(d) $\arccos(\cos(3\pi))$

We know $\cos(3\pi) = \cos(\pi) = -1$. So $\arccos(\cos(3\pi)) = \arccos(-1) = \pi$ (for the reasoning on the last step, see part (a)).

3. Show that $\cot\theta = \dfrac{1+\cos 2\theta}{\sin 2\theta}$ is an identity. Hint: use the double angle identities $\cos 2\theta = \cos^2\theta - \sin^2\theta$ and $\sin 2\theta = 2\sin\theta\cos\theta$.

$$\frac{1+\cos 2\theta}{\sin 2\theta} = \frac{1+\cos^2\theta-\sin^2\theta}{2\sin\theta\cos\theta}.$$

Using the fact that $1-\sin^2\theta = \cos^2\theta$, we get

$$\begin{aligned}\frac{1+\cos^2\theta-\sin^2\theta}{2\sin\theta\cos\theta} &= \frac{2\cos^2\theta}{2\sin\theta\cos\theta}\\ &= \frac{\cos\theta}{\sin\theta}\\ &= \cot\theta.\end{aligned}$$

4. Find all solutions to the following equations

(a) $\sin x = \frac{1}{2}$

(b) $2\sin^2 x - 3\sin x + 1 = 0$

(a) From our knowledge of the unit circle, we know that the solutions in the interval $[0, 2\pi)$ are $\pi/6$ and $5\pi/6$. Since sin has period 2π, all solutions are given by $\pi/6 + 2\pi k$ or $5\pi/6 + 2\pi k$ where k is an integer.

(b) Let $y = \sin x$. This equation becomes $2y^2 - 3y + 1 = 0$. Factoring gives $(2y - 1)(y - 1) = 0$, which means $y = 1$ or $y = \frac{1}{2}$. That is, $\sin x = 1$ or $\sin x = \frac{1}{2}$. We know $\sin x = 1$ exactly when $x = \pi/2 + 2\pi k$ (where k is an integer), and from part (a) $\sin x = \frac{1}{2}$ when $x = \pi/6 + 2\pi k$ or $5\pi/6 + 2\pi k$. These three sets of numbers form the solution to the equation.

5. Is it true that $\cos(x) \cdot \cos^{-1}(x) = 1$ is an identity? Why, or why not?