Math 32, Spring 2010, Section 101
Worksheet 2 Solutions

Work through the following problems in groups of about three. Take turns writing; everyone should get a chance to write for some of the problems. It's more important to understand the problems than to do all of them.

1. Evaluate or simplify each expression:

   (a) $4 + |-4| = 8$
   (b) $2 - |-2| = 0$
   (c) $|-2 + 4| = 2$
   (d) $||-7| - |-9|| = 2$
   (e) $\left|1 - \sqrt{2}\right| + 1 = \sqrt{2}$
   (f) $\left|-\sqrt{3} + \sqrt{5}\right| = -\sqrt{3}+\sqrt{5}$ (because $-\sqrt{3}+\sqrt{5}$ is a positive number, it is unchanged by taking absolute value.)

2. Rewrite each expression using absolute value notation:

   (a) The distance between x and 2 is at least 3/4.
   $|x - 2| \geq 3/4$
   (b) The number y is less than 3 units from the origin
   $|y| < 3$
   (c) The sum of the distances of a and b from the origin is greater than or equal to the distance of $a + b$ from the origin.
   $|a| + |b| \geq |a + b|$

3. Solve each equation

   (a) $2m - 1 + 3m + 5 = 6m - 8$ is equivalent to $m = 12$.
   (b) $(x - 2)(x + 1) = x^2 + 11$ is equivalent to $x^2 - x - 2 = x^2 + 11$ is equivalent to $x = -13$.
   (c) **[note: typo in original problem has been changed]** $x^3 + x^2 - 6x = 0$ is equivalent to $x(x^2 + x - 6) = 0$ is equivalent to $x(x - 2)(x + 3) = 0$ which has solutions $x = 0$, $x = 2$ and $x = -3$.

(d) $y + 3 + \frac{2}{y-1} = \frac{2y}{y-1}$. Clearing denominators we get $(y+3)(y-1)+2 = 2y$, or $y^2 + 2y - 3 + 2 = 2y$ which is equivalent to $y^2 - 1 = 0$. Factoring we get $(y-1)(y+1) = 0$, which has solutions $y = 1$ and $y = -1$. However, in clearing the denominator we multiplied by an expression involving y, so we have to check for extraneous solutions. Since we divide by $y - 1$ in the original equation, $y = 1$ is not a valid solution. However, one can check that $y = -1$ is a valid solution.

4. A triangle in the Cartesian plane has vertices at coordinates $(1, 4)$, $(5, 3)$ and $(3, 1)$. What are the lengths of the sides of the triangle? Is it a right triangle?

   Using the distance formula, we get that the sides have lengths $\sqrt{(1-5)^2 + (4-3)^2}$, $\sqrt{(5-3)^2 + (3-1)^2}$ and $\sqrt{(1-3)^2 + (4-1)^2}$. Simplified, that's $\sqrt{17}$, $\sqrt{8}$ and $\sqrt{13}$. It is a right triangle if and only if the sums of the squares of the lengths of the smaller sides is the length of the longer side. However, $\sqrt{8}^2 + \sqrt{13}^2 \neq \sqrt{17}$, so it is not a right triangle.

5. Write equations for the following lines:

   (a) The line through $(3, 5)$ and $(5, 11)$. The slope is $(11 - 5)/(5 - 3) = 3$. Using the point-slope formula from page 50 of the book, we get the equation $y - 5 = 3(x - 3)$ (or alternately $y - 11 = 3(x - 5)$).

   (b) The line through $(10, 9)$ and $(12, 9)$. Here, the two y-coordinates are the same, so we have the line $y = 9$.

   (c) The line through $(2, 2)$ that is parallel to the line $y = 7x + 13$. Since our line is parallel to $y = 7x + 13$, it means that the two lines have the same slope. Since the given line is in slope-intercept form (see p.50), we can see that this slope is 7. So the line we want has slope 7, and goes through $(2, 2)$. Using point-slope form, this gives $y - 2 = 7(x - 2)$.

   (d) The line through $(2, 2)$ that is perpendicular to the line $y = 7x + 13$. From p.50, lines perpendicular to one with a slope of 7 have a slope of $-\frac{1}{7}$ (the opposite of the reciprocal). Thus one answer is $y - 2 = -frac17(x - 2)$.

6. You may have seen the triangle inequality $|a + b| \leq |a| + |b|$, which is true for all numbers a and b. For which values of a and b do we have $|a + b| = |a| + |b|$? Justify your answer.

   We will have $|a + b| = |a| + |b|$ when a and b have the same sign. Play around with some specific choices of numbers to convince yourself that this is true.

7. Suppose r_1 and r_2 are the two real roots of $x^2 - 10x + 15$. What is their sum $r_1 + r_2$? How about their product $r_1 r_2$? (Hint: you don't need to find r_1 or r_2).

   When we factor $x^2 - 10x + 15 = (x - r_1)(x - r_2)$, we have $x^2 - 10x + 15 = x^2 - (r_1 + r_2)x + r_1 r_2$. Matching coefficients, we get $r_1 + r_2 = 10$ and $r_1 r_1 = 15$. This is a backwards version of the trick we use when factoring. That is, if you were to factor something like $x^2 - x - 6$, you would look for numbers that add to 1 and multiply to 6, and these would be the roots. You're using the fact that the roots multiply to the constant term, and add to negative the coefficient of x.

8. (a) Suppose I have two lines, $y = mx + b$ and $y = nx + c$. In terms of m,n,b and c, where do the two lines intersect?
   The x-coorindate of the intersection is when $mx + b = nx + c$ (when the two y-values are the same). Grouping like terms and factoring gives $(m - n)x = c - b$, which is equivalent to $x = (c - b)/(m - n)$. The y-coordinate of the intersection is the y-coordinate of (both!) lines at this x-value. Plugging in, we get two different looking, but equal, expressions $m(c - b)/(m - n) + b$ and $n(c - b)/(m - n) + c$.

   (b) If the two lines are parallel, they don't intersect. Why doesn't that contradict your answer to part (a)?

   If two lines are parallel, they have the same slope, so $m = n$. Since we had to divide by $m - n$ above, that method won't work to find the intersection point. This is a good thing; there is no intersection point!

9. Consider the line segment joining the points $P(2, 3)$ and $Q(6, 5)$. Find the equation of the line perpendicular to the line segment $\overline{PQ}$ that goes through its midpoint.

   Using the midpoint formula from the book, we get that the midpoint is $((2+6)/2, (3+5)/2)$, which is $(4, 4)$. The slope of the line segment is given by $\Delta y/\Delta x$, which is $2/4$ or $\frac{1}{2}$. Thus the slope of the perpendicular line is the opposite of the reciprocal, -2. So we want the line with slope -2 that goes through $(4, 4)$, which is given by point-slope form as $y - 4 = -2(x - 4)$.