Math 32, Spring 2010, Section 101
Worksheet 4 Solutions

Work through the following problems in groups of about four. Take turns writing; everyone should get a chance to write for some of the problems. It's more important to understand the problems than to do all of them.

1. Solve the inequality, and write the answer using interval notation.

   (a) $1 - 2(t+3) - t \leq 1 - 2t$

   Expanding gives $-5 - 3t \leq 1 - 2t$. Adding $3t$ to both sides gives $-5 \leq 1 + t$, and subtracting 1 from both sides gives $t \geq -6$. In interval notation, that's $[6, \infty)$.

   (b) $-3 \leq 2x + 1 \leq 5$

   Adding -1 to all of the terms we get $-4 \leq 2x \leq 4$, and dividing by two gives $-2 \leq x \leq 2$. In interval notation, that's $[-2, 2]$.

   (c) $|3x + 5| > 17$

   We have $|3x + 5| > 17$ when $3x+5 > 17$ or $3x+5 < -17$. The first inequality has solutions $x > 4$, and the second has solutions $x < -22/3$. Putting these together, we get a solution $(-\infty, -22/3) \cup (4, \infty)$.

   (d) $x^2 - 2x \leq 3$

   Rearranging gives $x^2 - 2x - 3 \leq 0$, or $(x-3)(x+1) \leq 0$. Thus the key numbers are 3 and -1, so we have to test a point in the intervals $(-\infty, -1)$, $(-1, 3)$, and $(3, \infty)$. Plugging in -2, we see that the inequality becomes false since $(-2-3)(-2+1)$ is not ≤ 0.) Plugging in 0 makes the inequality true, so $[-1, 3]$ is part of the solution (because $(-1, 3)$ was the interval the test point 0 was in, and we included the end points because the original inequality was "$\leq$" not "$<$"...alternatively, one can check that plugging in -1 and 3 make the inequality true, so they must be in the solution). Finally, plugging in 4 we see that the last interval is not part of the solution. So the solution set is $[-1, 3]$.

(e) $\frac{x^2-8x-9}{x} < 0$
Equivalently, $\frac{(x-9)(x+1)}{x} < 0$, which has key numbers -1, 0 and 9. We will test the points -2, $-\frac{1}{2}$, 1, and 10. One can check that the inequality is true for the first and third points, so our solution is $(-\infty, -1) \cup (0, 9)$. None of the endpoints are included, because none of them make the inequality true.

(f) $\frac{x+1}{x+2} > \frac{x-3}{x+4}$

This is equivalent to $\frac{x+1}{x+2} - \frac{x-3}{x+4} > 0$, or finding a common denominator,

$$\frac{(x+4)(x+1) - (x+2)(x-3)}{(x+2)(x+4) > 0}.$$

Expanding the numerator we get $\frac{6x+10}{(x+2)(x+4)} > 0$. So the key numbers are -4, -2, and $-5/3$ (which is between -1 and -2). Testing -5, -3, -1.9 and 0 we get that the solution is $(-4, -2) \cup (-\frac{5}{3}, \infty)$.

2. Given a person x, let $f(x)$ be x's sister's age. Is f a function? Why or why not?

   This is not actually a function. For one, f is undefined if x does not have any sisters. However, we could think of the domain of f as being people who have one sister. Even in that case, f is not a function, because f gives more than one value if x has more than one sister.

3. Give an example of a function whose domain is UC Berkeley students. What can you say about the range?

   E.g. If x is a UC Berkeley student, let $f(x)$ be the number of units that x is enrolled in. The range of this function will be whole numbers $1, 2, 3, 4, \ldots$ up to the largest number of units any student is currently enrolled in.

4. Determine the domain of each of the following functions

(a) $f(t) = \sqrt{\frac{2-t}{t+4}}$

The function is undefined when $t = -4$, so -4 is not in the domain. The function is also only defined when $\frac{2-t}{t+4} \geq 0$. This inequality has key numbers -4 and 2. Testing -10, 0 and 10 we see that the inequality holds for t in $(-4, 2]$. (Note: the interval is half-open and half-closed because 2 makes the inequality true, but -4 doesn't because the resulting expression is undefined). So the domain is the set of points $(-4, 2]$.

(b) $h(x) = \frac{x+1}{\sqrt{x+6}-x}$

This function will be undefined when $\sqrt{x+6} - x = 0$. That is, when $\sqrt{x+6} = x$. Squaring both sides, we get $x + 6 = x^2$, or $x^2 - x - 6 = 0$. Factoring gives $(x - 3)(x + 2) = 0$, or $x = 3$ and $x = -2$. Since we squared both sides, we need to check for extraneous solutions. Plugging in, we see that $\sqrt{3+6} - 3 = 0$, but $\sqrt{-2+6} - (-2) \neq 0$. So the denominator is only 0 when $x = 3$.

The function is also only defined when $x + 6 \geq 0$, i.e. when $x \geq -6$. So the domain is the interval $[-6, \infty)$ excluding the point 3. Another way of writing this is $[-6, 3) \cup (3, \infty)$.

5. Find the domain and range of the function $k(x) = \frac{2x-7}{3x+24}$.

Setting $y = \frac{2x-7}{3x+24}$, we solve for x and find the domain of the resulting function. Clearing the denominator we get $y(3x+24) = 2x-7$, or $3xy+24y = 2x-7$. Rearrange to gather all x terms: $3xy - 2x = -24y - 7$. Factor out an x: $x(3y - 2) = -24y - 7$, and then divide to get $x = \frac{-24y-7}{3y-2}$. The domain of this function is all numbers exlucding 2/3, so that is the range of the original function. In interval notation, $(-\infty, 2/3) \cup (2/3, \infty)$.

6. Write an inequality whose solutions are the values of k such that $x^2 - 2kx + 4$ has 2 real solutions. You don't have to solve it.

This quadratic has 2 real solutions when the discriminant is positive. That is, when $(2k)^2 - 4 * 4 > 0$. That is, when $4k^2 - 16 > 0$.