Math 32, Spring 2010, Section 101
Worksheet 7

Work through the following problems in groups of about four. Take turns writing; everyone should get a chance to write for some of the problems. It's more important to understand the problems than to do all of them.

1. True or false? Correct any false statements.

   (a) $\ln(x+y) = \ln(x) + \ln(y)$

   (b) $\ln(\sqrt{e}) = \frac{1}{2}$

   (c) The range of $\ln x$ is all real numbers.

   (d) If $a = b^c$, then $\log_c(b) = a$.

   (a) False. $\ln(x \cdot y) = \ln(x) + \ln(y)$.

   (b) True. $\ln\sqrt{e} = \ln e^{\frac{1}{2}} = \dfrac{1}{2}$.

   (c) True.

   (d) False. If $a = b^c$, then $\log_b(c) = a$. This is the definition of the logarithm.

2. Find the domain of each of the following functions.

   (a) $y = (\ln x)^2$

   (b) $y = \ln(x^2)$

   (c) $y = \ln(2 - x - x^2)$

   (d) $y = \log_3(e^x - 1)$

   (a) $\ln x$ is only defined when $x > 0$, so the domain is $(0, \infty)$.

   (b) This is defined when $x^2 > 0$. That is, when $x \neq 0$. Hence the domain is $(-\infty, 0) \cup (0, \infty)$.

   (c) This is only defined when $2 - x - x^2 > 0$. Multiplying by -1, we get $x^2 + x - 2 < 0$. Factoring yields $(x+2)(x-1) < 0$. Using the method of key numbers, we get that this equality holds when $-2 < x < 1$. Thus our domain is $(-2, 1)$.

   (d) This is defined when $e^x - 1 > 0$. Equivalently, when $e^x > 1$. This is true when $x > 0$. Thus the domain is $(0, \infty)$.

3. Solve the equation $\log_6 x + \log_6(x+1) = 0$.

   Combining the left side, we get $\log_6(x(x+1)) = 0$. We now raise 6 to the power of each side. The right-side becomes $6^0 = 1$. Thus the equation becomes

$$\begin{aligned} 1 &= 6^0 \\ &= 6^{\log_6(x(x+1))} \\ &= x(x+1). \end{aligned}$$

   Expanding, this is $x^2 + x = 1$, or $x^2 + x - 1 = 0$. The quadratic formula gives

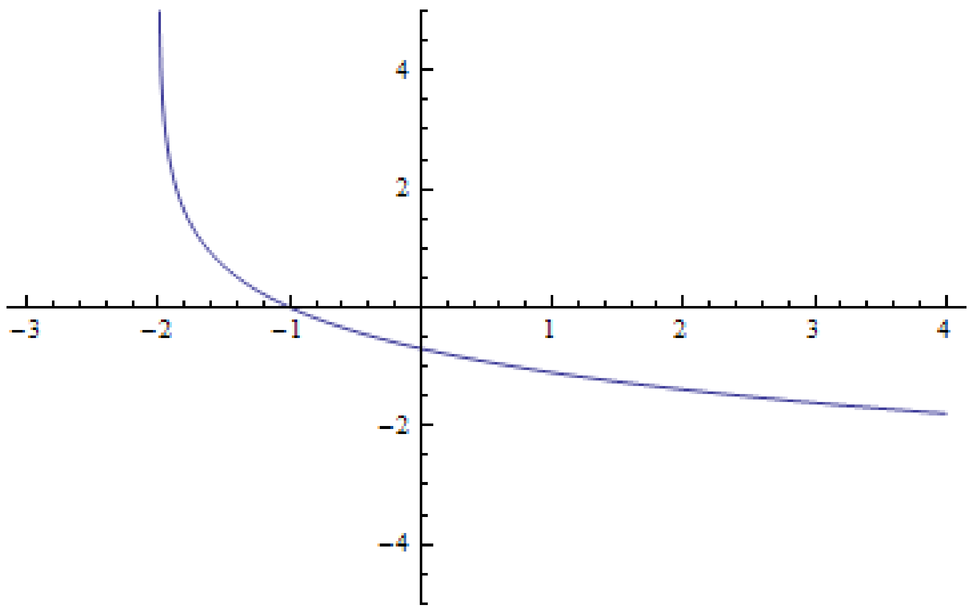
$$x = \frac{-1 \pm \sqrt{5}}{2}.$$

   However, looking at the original equation, we can see that we need $x > 0$ (and $x > -1$, but this is satisfied when $x > 0$) for the equation to make sense. Since $-1 - \sqrt{5}$ is negative, but $-1 + \sqrt{5}$ is positive, we get exactly one solution, namely $\frac{1}{2}(-1 + \sqrt{5})$.

4. Solve the inequality $\ln x + \ln(x+2) \leq \ln 35$.

   First, observe that the domain of the left-hand side is $x > 0$, so our answer must be contained within that interval. We now combine to get $\ln(x(x+2)) \leq \ln 35$. We can exponentiate both sides to get $x(x+2) \leq 35$, or $x^2 + 2x - 35 \leq 0$. Factoring the left, we get $(x+7)(x-5) \leq 0$. Using the method of key numbers, we can get that this is true when $-7 \leq x \leq 5$. However, we also need $x > 0$ from before, so the answer is $(0, 5]$.

5. Graph the function $y = -\ln(x+2)$, and specify any asymptotes and intercepts. What is the inverse of this function?



It has a vertical asymptote at $x = -2$. The y-intercept is found by plugging in $x = 0$, which gives $(0, -\ln 2)$. The x-intercept occures when $0 = -\ln(x + 2)$. Multiplying through by -1 and then exponentiating gives $1 = x + 2$, or $x = -1$.