Name: Solution

Math 54, Spring 2009, Section 109
Quiz 2

(1) (3 pts) Find $\det \begin{bmatrix} 1 & 0 & 3 & 0 \\ 4 & 2 & -2 & 0 \\ 6 & 0 & 1 & 0 \\ 5 & 4 & 3 & -1 \end{bmatrix}$.

$$\begin{vmatrix} 1 & 0 & 3 & 0 \\ 4 & 2 & -2 & 0 \\ 6 & 0 & 1 & 0 \\ 5 & 4 & 3 & -1 \end{vmatrix} = -\begin{vmatrix} 1 & 0 & 3 \\ 4 & 2 & -2 \\ 6 & 0 & 1 \end{vmatrix} = -2\begin{vmatrix} 1 & 3 \\ 6 & 1 \end{vmatrix} = 34$$

expand across last column

expand across second column

(2) (a) (2 pts) What is the definition of a linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ that is one-to-one? What is the definition of a linear transformation that is onto?

See solution for section 112.

(b) (1 pt) Choose one of the above properties (one-to-one, or onto), and state an equivalent property of the standard matrix of T. (e.g. "T is one-to-one if and only if the standard matrix of T has 47 rows," but something true...)

(3) (a) (2pts) Let H be the subspace of $\mathbb{R}^3$ with ordered basis $\mathfrak{B} = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix} \right\}$. Given that $\vec{x} = \begin{bmatrix} -1 \\ 4 \\ 6 \end{bmatrix}$ is in H, find the coordinates of $\vec{x}$ with respect to $\mathfrak{B}$.

Want c_1, c_2 such that $c_1 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix} = \vec{x}$, i.e. $\begin{bmatrix} 1 & -1 \\ 2 & 2 \\ 3 & 3 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \\ 6 \end{bmatrix}$.

so row reduce $\begin{bmatrix} 1 & -1 & -1 \\ 2 & 2 & 4 \\ 3 & 3 & 6 \end{bmatrix} \to \begin{bmatrix} 1 & -1 & -1 \\ 0 & 4 & 6 \\ 0 & 6 & 9 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & 3/2 \\ 0 & 0 & 0 \end{bmatrix}$,

so $c_1 = \frac{1}{2}$, $c_2 = 3/2$. Thus $[\vec{x}]_{\mathfrak{B}} = \begin{bmatrix} \frac{1}{2} \\ 3/2 \end{bmatrix}$.

(b) (1 pt) Suppose $T : \mathbb{R}^3 \to \mathbb{R}$ is a linear transformation with

$$T\left(\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\right) = -2, \qquad T\left(\begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}\right) = 2.$$

Find $T\left(\begin{bmatrix} -1 \\ 4 \\ 6 \end{bmatrix}\right)$.

$$T\left(\begin{bmatrix} -1 \\ 4 \\ 6 \end{bmatrix}\right) = T\left(\frac{1}{2}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + \frac{3}{2}\begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}\right) = \frac{1}{2} T\left(\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\right) + \frac{3}{2} T\left(\begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}\right) =$$

$$= \frac{1}{2} \cdot -2 + \frac{3}{2} \cdot 2 = 2$$