Name: ______________________________

**Math 54, Spring 2009, Section 109**
**Quiz 6**

[**1** - (5 pts)] Solve the initial value problem

$$y''' - y'' - 4y' + 4y = 0,$$
$$y(0) = -4, \quad y'(0) = -1, \quad y''(0) = -19.$$

The auxiliary equation is $r^3 - r^2 - 4r + 4 = 0$. By inspection, we can see that $r = 1$ is a root. Using polynomial long division (or more guess and check), we get $r^3 - r^2 - 4r + 4 = (r-1)(r-2)(r+2)$. So the general solution of this equation is

$$y(t) = c_1 e^t + c_2 e^{2t} + c_3 e^{-2t}.$$

Plugging in the initial conditions gives

$$\begin{aligned} -4 &= y(0) = c_1 + c_2 + c_3, \\ -1 &= y'(0) = c_1 + 2c_2 - 2c_3, \\ -19 &= y''(0) = c_1 + 4c_2 + 4c_3. \end{aligned}$$

To solve this system, we row reduce

$$\begin{bmatrix} 1 & 1 & 1 & -4 \\ 1 & 2 & -2 & -1 \\ 1 & 4 & 4 & -19 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & -2 \end{bmatrix}.$$

So $c_1 = 1$, $c_2 = -3$ and $c_3 = -2$, which yields the solution

$$y(t) = e^t - 3e^{2t} - 2e^{-2t}.$$

[**2** -(2 pts)] Write the following system of equations as a matrix system in normal form

$$\begin{aligned} &x'(t) - \sin(t)x(t) + e^t y(t) = 0 \\ &y'(t) - \cos(t)x(t) + (a + bt^3)y(t) = 0. \end{aligned}$$

Rearranging, we get the system

$$\begin{aligned} x'(t) &= \sin(t)x(t) - e^t y(t), \\ y'(t) &= \cos(t)x(t) - (a + bt^3)y(t). \end{aligned}$$

That is,

$$\begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix} = \begin{bmatrix} \sin(t)x(t) - e^t y(t) \\ \cos(t)x(t) - (a + bt^3)y(t) \end{bmatrix} = x(t) \begin{bmatrix} \sin(t) \\ \cos(t) \end{bmatrix} + y(t) \begin{bmatrix} -e^t \\ -(a + bt^3 \end{bmatrix} = \begin{bmatrix} \sin(t) & -e^t \\ \cos(t) & -a - bt^3 \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}.$$

So we get the system

$$\vec{w}'(t) = \begin{bmatrix} \sin(t) & -e^t \\ \cos(t) & -a - bt^3 \end{bmatrix} \vec{w}(t).$$

[**3** - (2 pts)] Express the given higher-order differential equation as a matrix system in normal form

$$my''(t) + by'(t) + ky(t) = 0,$$

where $m, b, k \in \mathbb{R}$ are constants.

This is equivalent to the system

$$\begin{aligned} y' &= v \\ v' &= -\tfrac{k}{m}y - \tfrac{b}{m}v, \end{aligned}$$

which leads to the normal form

$$\vec{x}'(t) = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix} \vec{x}(t).$$