Math 54, Spring 2009, Sections 109 and 112
Worksheet 1: Lay 1.1 - 1.4

(1) Which of the following matrices are in an echelon form? Reduced echelon form? If the matrix is in an echelon form, identify the pivots.

$$(a)\begin{pmatrix}1 & 0 & 1 & 4\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix} \qquad (b)\begin{pmatrix}2 & 2 & 1 & -2\\ 0 & 0 & 1 & 3\\ 0 & 0 & 0 & 1\end{pmatrix} \qquad (c)\begin{pmatrix}0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}$$

(2) (a) Does the following matrix equation have a solution?

$$\begin{pmatrix}1 & 2 & 3\\ 4 & 2 & -4\\ 1 & 1 & 1\end{pmatrix}\vec{x} = \begin{bmatrix}4\\ 0\\ 2\end{bmatrix}.$$

(b) Is $(4, 0, 2)$ in $\text{Span}\,\{(1, 4, 1), (2, 2, 1), (3, -4, 1)\}$? (Recall that for convenience we sometimes write vectors horizontally).

(3) Consider the set of vectors

$$S = \left\{ \begin{bmatrix} 2x_2 - x_1 \\ \pi x_3 + \sqrt{2}x_1 - x_2 \\ x_1 + x_2 + x_3 \end{bmatrix} : x_1, x_2, x_3 \in \mathbb{R} \right\}.$$

Can you write S as the span of a collection of vectors?

(4) Suppose $\vec{v}_1, \ldots, \vec{v}_p$ are vectors in $\mathbb{R}^n$ and let $V = \text{Span}\{\vec{v}_1, \ldots, \vec{v}_p\}$. Prove that if $\vec{x} \in V$ and $\vec{y} \in V$, then $\vec{x} + \vec{y} \in V$ as well.