Math 54, Spring 2009, Sections 109 and 112
Worksheet 2 (Lay 1.7-1.8)

(1) Classify the following sets as linearly independent or linearly dependent. (Hint: no calculations needed).

(a) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \right\}$.

(b) $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 5 \\ 6 \end{bmatrix} \right\}$.

(c) $\left\{ \begin{bmatrix} 1 \\ -2 \\ 3 \\ -4 \end{bmatrix}, \begin{bmatrix} -3 \\ -6 \\ 9 \\ -12 \end{bmatrix} \right\}$.

(2) True/False: If it's true, give a justification. If it's false, give a counterexample.

(a) If $\{\vec{v}_1, \ldots, \vec{v}_p\}$ is a linearly independent set of vectors, and A is a matrix, then $\{A\vec{v}_1, \ldots, A\vec{v}_p\}$ is also linearly independent.

(b) If $\vec{b}$ is in the span of the columns of A, then $A\vec{x} = \vec{b}$ is consistent.

(3) (#39 from p.72) Suppose A is a $m \times n$ matrix with the property that for all $\vec{b}$ in $\mathbb{R}^m$ the equation $A\vec{x} = \vec{b}$ has at most one solution. Explain why the columns of A are linearly independent.

(4) Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation, and let $\{\vec{v}_1, \ldots \vec{v}_p\}$ be a linearly dependent set in $\mathbb{R}^n$. Assume that $T(\vec{v}_i) \neq T(\vec{v}_j)$ when $i \neq j$. Show that $\{T(\vec{v}_1), \ldots, T(\vec{v}_p)\}$ is linearly dependent in $\mathbb{R}^m$. (Compare to 2a).