Math 54, Spring 2009, Sections 109 and 112
Worksheet 4 (Lay 4.1-4.3)

(1) Let V be the vector space of continuous functions from $\mathbb{R}$ to $\mathbb{R}$. Is the set $\{\sin x, \cos x, e^x\}$ linearly independent? Find a basis for $\text{Span}\{\sin x, \cos x, e^x\}$.

(2) True or False? If true, justify. If false, give a counterexample. In these statements, V is a vector space, and H is a subspace of V.

(a) If $\vec{u} \in H$ and $\vec{v} \in H$, then $\text{Span}\{\vec{u}, \vec{v}\} \subseteq H$.

(b) A basis for $\mathbb{P}_n$ (polynomials of degree at most n) has n elements.

(c) If a finite set S of non-zero vectors spans V, then some subset of S is a basis for V.

(d) A linear transformation is one-to-one if and only if $\text{Kernel}(T) = \{0\}$.

(3) Let $M_{n\times m}(\mathbb{R})$ be the vector space of $n \times m$ matrices. Define $T : M_{2\times 3}(\mathbb{R}) \to M_{2\times 3}(\mathbb{R})$ by $T(A) = AB$, where $B = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 6 \\ 0 & 4 & 5 \end{bmatrix}$ is fixed. Show that T is one-to-one and onto (i.e. find Range(T) and Kernel(T)).

(4) Let V be the vector space of continuous functions from $\mathbb{R}$ to $\mathbb{R}$ that also have a continuous derivative, and let W be the vector space of continuous functions from $\mathbb{R}$ to $\mathbb{R}$. Define $T : V \to W$ by $T(f) = f'$. Justify why V and W are vector spaces, and why T is a linear transformation. What is $\ker T$? Bonus: use calculus to show that T is onto.