Math 54, Spring 2009, Sections 109 and 112
Worksheet 5 (Lay 4.5-4.7)

(1) (p.276, #6) Let $\mathcal{D} = \{\vec{d}_1, \vec{d}_2, \vec{d}_3\}$ and $\mathcal{F} = \{\vec{f}_1, \vec{f}_2, \vec{f}_3\}$ be bases for a vector space V, and suppose $\vec{f}_1 = 2\vec{d}_1 - \vec{d}_2 + \vec{d}_3$, $\vec{f}_2 = 3\vec{d}_2 + \vec{d}_3$, and $\vec{f}_3 = -3\vec{d}_1 + 2\vec{d}_3$. Find the change-of-coordinate matrix from $\mathcal{F}$ to $\mathcal{D}$. Find $[\vec{x}]_{\mathcal{D}}$ for $\vec{x} = \vec{f}_1 - 2\vec{f}_2 + 2\vec{f}_3$.

(2) True or False? If true, justify. If false, give a counterexample.

(a) If $\mathcal{B}$ and $\mathcal{C}$ are different bases for V, then $\underset{\mathcal{C}\leftarrow\mathcal{B}}{P}$ can be singular.

(b) Let H be a subspace of a finite-dimensional vectors space V, and let $\mathcal{B} = \{b_1, \ldots, b_r\}$ be a basis for V. Then $H = V$ if and only if $\mathcal{B} \subset H$.

(c) If P is an invertible $n \times n$ matrix, then there are bases $\mathcal{B}$ and $\mathcal{C}$ for $\mathbb{R}^n$ such that $P = \underset{\mathcal{C}\leftarrow\mathcal{B}}{P}$.

(3) Let A be an $n \times n$ matrix, and let $\mathcal{B} = \{\vec{b}_1, \ldots, \vec{b}_n\}$ be a basis for $\mathbb{R}^n$. Find a formula for the matrix C such that $C[\vec{x}]_{\mathcal{B}} = [A\vec{x}]_{\mathcal{B}}$.

(4) (p. 299, # 9) Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation. What are the dimensions of the range and kernel of T if T is one-to-one? What about if T is onto?