Math 54, Summer 2009, Lecture 4
Midterm 1 Review Exercises

These exercises don't cover some of the **very important** computational-type problems, including many of the things listed under the "be able to" section of the review sheet. You can find examples of those types of problems on the sample exam and in the sections of the book (including the supplemental exercises at the end of each chapter). These are a little more theoretical, and are aimed at making sure you have a good grasp of the ideas underlying the algorithms.

1) Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation, and A be its standard matrix. Complete the following table so that statements in the same row are equivalent:

Property of T	Columns of A	Pivots of A	$A\vec{x} = \vec{b}$?
One-to-one			≤ 1 solution for every $\vec{b}$
	Span $\mathbb{R}^m$		

2) Let A be a 17×17 matrix such that $A^{12} = I_{17}$. Explain why $A\vec{x} = \vec{0}$ only when $\vec{x} = \vec{0}$.

3) (#10, p.184) Suppose A is an invertible square matrix. Explain why $A^T A$ is also invertible, and then show that $A^{-1} = (A^T A)^{-1} A^T$.

4) Suppose you have a square matrix such that $A^3 = 0$ (the zero matrix). Use matrix algebra to compute $(I - A)(I + A + A^2)$. Generalize to show that if $A^k = 0$ for some $k \geq 1$, then $(I - A)$ is invertible.

5) True or False? If true, justify. If false, provide a counterexample. (Some of these are from p.102.)

(a) If $\{\vec{v}_1, \vec{v}_2\}$ is a linearly independent set in $\mathbb{R}^n$, so is $\{\vec{v}_1, \vec{v}_1 + \vec{v}_2\}$.

(b) If an $m \times n$ matrix A has a pivot in every column or has a pivot in every row, then it is invertible.

(c) If T is a linear transformation, then $T(\vec{0}) = \vec{0}$.

(d) If A is a square matrix, then it can be written as a product of elementary matrices.

(e) If A is an $n \times n$ matrix such that $A\vec{x} = \vec{b}$ is consistent for every $\vec{b}$, then A has a pivot in every column.

Bonus: **(6)** Let $A = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$. (a) Factor A^{-1} as a product of elementary matrices. That is, find elementary matrices $E_1, \ldots, E_m$ such that $A^{-1} = E_1 \cdots E_m$. (b) Use this to factor A as a product of elementary matrices.