

Name: Solution

Math 54, Summer 2009, Lecture 4  
Quiz 1

(1) Find the general solution of the following system of linear equations, or show that it is inconsistent.

$$\begin{aligned}x_1 + 3x_2 - 2x_3 &= 5 \\-4x_2 + 12x_3 &= -8 \\3x_1 + x_2 + 18x_3 &= -1\end{aligned}$$

Augmented matrix

$$\begin{bmatrix} 1 & 3 & -2 & 5 \\ 0 & -4 & 12 & -8 \\ 3 & 1 & 18 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & -2 & 5 \\ 0 & -4 & 12 & -8 \\ 0 & -8 & 24 & -16 \end{bmatrix} \xrightarrow{\text{Scale rows 2 and 3}} \begin{bmatrix} 1 & 3 & -2 & 5 \\ 0 & 1 & -3 & 2 \\ 0 & 1 & -3 & 2 \end{bmatrix} \sim$$

Reduced Echelon Form

$$\sim \begin{bmatrix} 1 & 3 & -2 & 5 \\ 0 & 1 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 7 & -1 \\ 0 & 1 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$x_3$  free

$$x_1 + 7x_3 = -1$$

$$x_2 - 3x_3 = 2, \quad \text{or}$$

$$\begin{cases} x_1 = -1 - 7x_3 \\ x_2 = 2 + 3x_3 \\ x_3 \text{ free} \end{cases}$$

(2) Say whether the given statement is true or false. If it is true, justify it. If it is false, provide a counterexample showing that it is false. "A linear system of 2 equations and 3 variables is always consistent."

False. 
$$\begin{cases} x_1 + x_2 + x_3 = 1 \\ x_1 + x_2 + x_3 = 47 \end{cases} \text{ is inconsistent.}$$

Can see this by inspection, or

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 47 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 46 \end{bmatrix}. \quad [0 \ 0 \ 0 \ b], \quad b \neq 0 \Rightarrow \text{inconsistent.}$$

(3) Give all values of  $h$  for which the following system has exactly one solution, for which it has no solutions, and for which it has many solutions.

$$x_1 + 2x_2 = -3$$

$$x_1 + 6x_2 = h$$

$$2x_1 + 6x_2 = -2$$

$$\begin{bmatrix} 1 & 2 & -3 \\ 1 & 6 & h \\ 2 & 6 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & -3 \\ 0 & 4 & h+3 \\ 0 & 2 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & -3 \\ 0 & 4 & h+3 \\ 0 & 0 & 4 - \frac{h}{2} - \frac{3}{2} \end{bmatrix}$$

Has no free variables, so never has many solutions.

When  $4 - \frac{h}{2} - \frac{3}{2} = 0$ , has 1 solution. That is, when  $h = 5$ .

When  $h \neq 5$ , it has no solution, because  $4 - \frac{h}{2} - \frac{3}{2} \neq 0$ .