Name: ______________________________

Math 54, Summer 2009, Lecture 4
Quiz 5 Solutions

(1) Determine if W is a subspace of $\mathbb{P}_3$. Justify your answer by proving its a subspace, or showing that it is not.

(a) W is all polynomials of the form $p(t) = ct^3$ for some $c \in \mathbb{R}$.

The zero polynomial $p(t) = 0$ is of the form ct^3 (with $c = 0$), and is therefore in W. If $p_1(t) = ct^3$ and $p_2(t) = dt^3$, then $(p_1 + p_2)(t) = (c + d)t^3$ is in W. Also, $(rp_1)(t) = rct^3$ is in W, so W is a subspace of $\mathbb{P}_3$. Easier way: $W = \text{Span}\{t^3\}$ and is therefore a subspace.

(b) W is all polynomials $p(t)$ in $\mathbb{P}_3$ such that $p(47) = 0$.

The zero polynomial $p(t) = 0$ certainly has $p(47) = 0$. If $p_1, p_2 \in W$, then $(p_1 + p_2)(47) = p_1(47) + p_2(47) = 0 + 0 = 0$, so $p_1 + p_2 \in W$. If $r \in \mathbb{R}$, then $(rp_1)(47) = rp_1(47) = r0 = 0$, so $rp_1 \in W$. Thus W is a subspace of P_3. Alternate approach: W is the kernel of the linear transformation $p \mapsto p(47)$, although you would have to prove that this is linear.

(c) W is all polynomials in $\mathbb{P}_3$ whose coefficients are all integers.

Not a subspace. The polynomial $p(t) = t^3 + 2 \in W$, but scaling by $1/3$ yields a polynomial that is not in W.

(2) Determine whether or not the set $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 9 \\ 12 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 4 \end{bmatrix} \right\}$ is a basis for $\mathbb{R}^3$.

Row redcuing we get

$$A = \begin{bmatrix} 1 & 4 & -1 \\ 2 & 9 & 1 \\ 3 & 12 & 4 \end{bmatrix} \to \begin{bmatrix} 1 & 4 & -1 \\ 0 & 1 & 3 \\ 0 & 0 & 7 \end{bmatrix}.$$

Since A has a pivot in every column, its columns are linearly independent. Since it has a pivot in every row, its columns span $\mathbb{R}^3$. Thus the columns of A are a basis for $\mathbb{R}^3$.

(3) Let $H = \operatorname{Span} \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 4 \\ 3 \\ 2 \\ 1 \end{bmatrix} \right\}$.

(a) Find a matrix A such that $H = \operatorname{Col} A$.

$$\begin{bmatrix} 1 & 4 \\ 2 & 3 \\ 3 & 2 \\ 4 & 1 \end{bmatrix}$$

(b) If possible, find a second matrix B such that $H = \operatorname{Col} B$. Otherwise, explain why it's impossible.

$$\begin{bmatrix} 4 & 1 \\ 3 & 2 \\ 2 & 3 \\ 1 & 4 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} 1 & 4 & 1 \\ 2 & 3 & 2 \\ 3 & 2 & 3 \\ 4 & 1 & 4 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} 1 & 8 \\ 2 & 6 \\ 3 & 4 \\ 4 & 2 \end{bmatrix}, \text{ etc.}$$