Math 54, Summer 2009, Lecture 4
Worksheet 2: Lay 4.1-4.2

(1) Let $A = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$. Describe how you would solve the following questions, but don't actually do the computation.

(a) Is $\begin{bmatrix} 5 \\ 5 \end{bmatrix} \in \operatorname{Col} A$?

(b) Is $\begin{bmatrix} 5 \\ 5 \end{bmatrix} \in \operatorname{Nul} A$?

(c) Is $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \in \operatorname{Col} A$?

(d) Is $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \in \operatorname{Nul} A$?

(e) Find something non-zero in $\operatorname{Nul} A$.

(f) Find something non-zero in $\operatorname{Col} A$.

(2) Let $W = C[a,b]$, the vector space of continuous functions $f : [a,b] \to \mathbb{R}$. Let $V = C^1[a,b]$, the vector space of differentiable functions $f : [a,b] \to \mathbb{R}$ whose derivative is continuous. Define $D : V \to W$ by $D(f) = f'$.

(a) Explan why it makes sense for $D : V \to W$, and show that D is a linear transformation.

(b) What is $\operatorname{Ker} D$?

(c) Bonus: show that D is onto.

(3) Let $V = M_{2\times 3}$, the vector space of 2×3 matrices with real entries. Assume that A is a 3×3 matrix, and that A is invertible. Define $T : V \to W$ by $T(B) = BA$.

(a) What should W be?

(b) Show that T is a linear transformation.

(c) What is $\operatorname{Ker} T$? What is $\operatorname{Ran} T$?