DARPA REPORT FOR SUBFACTORS IN MAUI
JULY 15-19, 2013

Schedule of talks

Monday, July 15.
Dietmar Bisch (Vanderbilt University):
Title: On infinite depth subfactors
Abstract: The first irreducible, infinite depth, hyperfinite subfactors appear at index 4. They have principal graphs A_∞ or D_∞, are (strongly) amenable, and hence classified by their standard invariant. The next non-trivial, irreducible infinite depth subfactor known has index $3+\sqrt{5}$ and its standard invariant is obtained as a free product of the standard invariants of an A_3 and an A_4 subfactor (in the sense of planar algebras - work of Jones and myself). By a result of Popa, it can be realized as a hyperfinite subfactor, but it is open how many non-isomorphic hyperfinite subfactors with this standard invariant exist.

If the Jones index is a composite integer, a construction of Popa, Nicoara and myself shows that there are uncountably many non-isomorphic, irreducible hyperfinite subfactors at that index with the same standard invariant. The construction relies heavily on the fact that there are many non-cocycle conjugate outer (cocycle) actions of property (T) groups on the hyperfinite II_1 factor.

Irreducible infinite depth subfactors with index between 4 and 5 have trivial (i.e. Temperley-Lieb) standard invariant and hence A_∞ principal graphs. They are typically constructed using a non-degenerate commuting square of multi-matrix algebras. A relative McDuff type theorem implies then that these subfactors have full relative fundamental group, and a large relative central sequence algebra.

We explain various invariants for subfactors, which supplement the standard invariant. These include the relative fundamental group, relative versions of property Γ, McDuff's property, property (T) etc. In particular, we prove that group-type subfactors have relative property Γ if and only if a certain infinite index subfactor obtained as a crossed product has it. This gives many examples of irreducible, hyperfinite, infinite depth subfactors, which do or do not have relative property Γ.

Dave Penneys (University of Toronto):
Title: Chirality and principal graph obstructions
Abstract: Determining which bipartite graphs can be principal graphs of subfactors is an important and difficult question in subfactor theory. Using only planar algebra techniques, we prove a triple point obstruction which generalizes all known initial triple point obstructions to possible principal graphs. We also prove a similar quadruple point obstruction with the same technique. A preprint of this work is available at arXiv:1307.5890.

David Jordan (UT Austin):
Title: Segal CFT: At the boundary
Abstract: We will introduce Graeme Segal's geometric description of a conformal field theory, and discuss the construction of the free fermion model. It is believed that Segal CFTs can be related to the operator algebraic notion of a conformal net of von Neumann algebras on the circle, but there is no general theory that makes this rigorous. However, we will discuss how

we are able to make this connection precise in the free fermion model, and obtain the fusion of representations of conformal nets as a "unitary boundary value" of the geometric theory.

Kevin Walker (Microsoft Station Q):
Title: Super planar algebras and D_{odd}
Abstract: If we consider planar algebras enriched in super vector spaces rather than vector spaces, then a version of D_{odd} exists. There also some super tadpoles. Similar techniques can be used to extend results on [de]equivariantization.

Scott Morrison (Australian National University):
Title: Categories generated by a trivalent vertex
Abstract: We explore all possible 'small' skein theories for trivalent graphs. We're limited by computing power (computing determinants and Groebner bases), but already we find some interesting examples, such as H3 in the Brauer-Picard groupoid of the Haagerup subfactor.

Tuesday, July 16.
Mike Hartglass (UC Berkeley):)
Title: Guionnet, Jones, and Shlyakhtenko C* algebras
Abstract: Given a planar algebra, P, Guionnet, Jones, and Shlyakhtenko created graded algebras associated to P. They showed that the associated von Neumann algebras form a Jones tower of II_1 factors whose standard invariant is P. This talk will focus on the C* algebras one can naturally associate to the aforementioned graded algebras. It will be shown that these algebras also form a Jones tower in an appropriate sense and that in many cases, their K-theory can be easily understood. One consequence of this talk is that even if P represents the standard invariant for the outer action of a finite group, the constructed tower of C* algebras does not encode the group action.

Benjamin Hayes (UCLA):
Title: Extended von Neumann Dimenstion for Representations of Equivalence Relations
Abstract: Last year, we defined extended von Neumann dimension for actions of sofic groups on Banach spaces. A particular application was to show that the action of G on $l^p(G)^{\oplus n}$ is not isomorphic to the action of G on $l^p(G)^{\oplus m}$ if n is not equal to m. We also used this to define l^p-Betti numbers of groups. In this talk, I will describe how one can define extended von Neumann dimension for representations of an equivalence relation. In particular, we define an analogue of "l^2-Betti number plus 1" for l^p-Homology, and show that it is between cost and the first l^2-Betti number plus 1. We also outline how this gives a plan of attack for solving cost versus l^2-Betti number.

Pinhas Grossman (University of New South Wales):
Title: Representation theory associated to finite-index subfactors, by example
Abstract: We illustrate the application of the categorified representation theory of fusion categories to finite-depth subfactors through several examples. In particular we show that the Asaeda-Haagerup subfactor is related to Izumi's Cuntz algebra subfactors and compute its quantum double. This is joint work with Noah Snyder and with Masaki Izumi.

Terry Gannon (University of Alberta):
Title: Near-group categories and their doubles
Abstract: I review my work with Evans, classifying and constructing quadratic extensions of group categories, and computing their doubles. This gives over 40 new finite depth subfactors with index between 6.8 and 14.9. We expect these to lie in infinite families, and their doubles to be realised by vertex operator algebras.

Noah Snyder (Indiana University):
Title: Small index subfactors: How strange are they?
Abstract: I'll begin by recapping this history of small index subfactors. Many small index subfactors are "well-understood" usually by a connection to groups or quantum groups. The Haagerup subfactor is not completely understood, but there is increasing evidence due to Izumi and Evans-Gannon that suggests that it should not be thought of as exception and lives in a discrete infinite family. In this talk I will discuss a new construction of the Asaeda-Haagerup subfactor which suggests that it too lives in an infinite discrete family closely related to the Haagerup family, and I will discuss some evidence that the extended Haagerup subfactor cannot be easily understood using any current techniques.

Wednesday, July 17.
Dimitri Shlyakhtenko (UCLA):
Title: Some Distributions of Noncommutative Random Variables
Abstract: In a joint work with P. Skoufranis we prove that selfadjoint polynomials of free semicircular variables have nonatomic distributions which are algebraic. The latter statement is also true for selfadjoint elements of grades algebras associated to planar algebras.

Corey Jones (Vanderbilt University):
Title: Affine annular Representations and the Drinfeld Center
Abstract: Given a (factor or subfactor) planar algebra, we define the affine annular category and consider Hilbert representations. By a result of Kevin Walker, this category is equivalent to the Drinfeld centre of the projection category of the planar algebra. Here we discuss a proof of this fact due to P. Das, S. Ghosh, and V. Gupta.

Sayan Das (Vanderbilt University):
Title: From Classical probability to free probability via cumulants
Abstract: In this talk we follow Jonathan Novak's approach to motivate free probability from classical probability. The use of cumulants is motivated by some simple problems in counting graphs, and leads to a very easy proof of the classical central limit theorem, which is mimicked to give a proof of the central limit theorem for free random variables.

Thursday, July 18.
David Evans (Cardiff University):
Title: Modular invariants and Twisted equivariant K-theory: KK-theory
Abstract: The twisted equivariant K-theoretic approach to modular invariants is discussed after a backdrop of the braided subfactor approach. The KK approach to modular invariants and conformal field theories, which is current work with Terry Gannon, is illustrated in the context of tori and lattices and as a potential framework for realising the double of the Haagerup subfactor and beyond whose modular data we have already shown to be understood through the double of a dihedral group and $SO(13)$ at level 2.

James Tener (UC Berkeley):
Title: Segal CFT: At the boundary
Abstract: We will introduce Graeme Segal's geometric description of a conformal field theory, and discuss the construction of the free fermion model. It is believed that Segal CFTs can be related to the operator algebraic notion of a conformal net of von Neumann algebras on the circle, but there is no general theory that makes this rigorous. However, we will discuss how we are able to make this connection precise in the free fermion model, and obtain the fusion of

representations of conformal nets as a "unitary boundary value" of the geometric theory.

Alice Guionnet (MIT):
Title: Free transport
Abstract: We will discuss the generalization of some aspects of classical transport theory to the non-commutative setting provided by free probability, and its application to prove isomorphisms of C^* and W^* algebras.

Zhengwei Liu (Vanderbilt University):
Title: Composed inclusions of an A_3 and an A_4 subfactor
Abstract: The study of composed inclusions of two subfactors was initiated by D. Bisch and U. Haagerup. We will discuss the classification in the case that the principal graphs of the two subfactors are A_3 and A_4. That answers a question posed by D. Bisch and U. Haagerup in 1994.

Friday, July 19.
Arnaud Brothier (KU Leuven):
Title: Subfactors with prescribed fundamental groups
Abstract: In a joint work with Stefaan Vaes we study fundamental groups for subfactors. We consider a large class of subgroups $\mathcal{S}$ of $\mathbb{R}_+^\times$ that contains all countable subgroups and some uncountable groups with any Hausdorff dimension between 0 and 1. Using Popa's deformation/rigidity theory, we construct a hyperfinite subfactor of index 6 with its fundamental group equal to G, for any group G in $\mathcal{S}$. Those subfactors are indexed by ergodic probability measure preserving transformation. This proves in particular that there are unclassifiably many subfactors at index 6 with the same standard invariant. Furthermore, using those technics we provide an explicit uncountable family of non outer conjugate actions on the hyperfinite II_1 factors for any non amenable group.

Jesse Peterson (Vanderbilt University):
Title: Rigidity for characters on lattices and commensurators
Abstract: A character on a discrete group is a conjugation invariant function of positive type which take value 1 at the identity. The study of characters on infinite groups was initiated by Thoma in 1964 where he classified extreme characters on the group of finite permutations of the natural numbers. Recently, Bekka has shown that characters on the groups $PSL_n(\mathbb{Z})$, for $n > 2$ have a remarkable rigidity property, they correspond either to finite dimension representations, or else the left regular representation. For the case $n = 2$, this was generalized to other rings of integers by the speaker and Thom. In this talk we will describe a similar rigidity property for characters on many irreducible lattices in higher rank semi-simple groups. This is related to rigidity results of Margulis, and Bader-Shalom, and confirms a conjecture of Connes for such groups. This is joint work with Darren Creutz.

Hans Wenzl (UC San Diego):
Title: On spinor tensor categories
Abstract: For this talk, a spinor category is a tensor category whose fusion semiring is isomorphic to the one of the representations of Spin(N) for N odd, or Pin(N), for N even. If S is the object corresponding to the generating spinor representation, we show that the endomorphisms of its n-th tensor power are given by a representation of a non-standard q-deformation $U_q'so_n$ of the universal enveloping algebra of the orthogonal Lie algebra so_n. This is the key result for proving that any spinor category is equivalent to one of finitely many twists of the representation category of a Drinfeld-Jimbo quantum group U_qso_N, or of a $\mathbb{Z}/2$ extension of it in the case of N even. Similar results hold for the associated finite fusion categories.

Yasuyuki Kawahigashi (University of Tokyo):
Title: Local conformal nets with trivial representation theory
Abstract: We construct a family of local conformal nets with trivial representation theory from triply even binary codes of lengths being multiples of 16. This is an operator algebraic counterpart of a recent construction of Lam-Yamauchi, but our method of the proof is entirely different from theirs. We also explain why such a construction is expected to be related to a general subfactor of finite depth.

Vaughan Jones (Vanderbilt University):
Title: A toy continuum limit for subfactor planar algebras
Abstract: We show how to assign nets of von Neumann algebras on the circle to any sub factor planar algebra by taking dyadic rationals on the circle. The diffeomorphism group is replaced by the Thompson group. For the very simplest planar algebra the Thompson group representation is that on ℓ^2(finite subsets of the dyadic rationals). One also obtains unitary representations of the Thompson group by fermionic second quantisation of the action on $L^2(S^1)$ using a polarization a la Segal coming from the wavelet basis of L^2.