(VOAs) (conformal nets)

Quantum fields from Haag-Kastler nets in 2D chiral conformal field theory

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MATRIX Workshop on 2D SUSY Theories + Related Topics

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A conformal net consists of:

- a Hilbert space $\mathcal{H}$, a vector $\Omega \in \mathcal{H}$
- a projective representation $U : \mathrm{Diff}_+(S^1) \circlearrowright H$
- von Neumann algebras $\mathcal{A}(I) \subset B(\mathcal{H})$ for all intervals $I \subset S^1$

closed under convergence in expectation

such that

- $I \subseteq J \Rightarrow \mathcal{A}(I) \subseteq \mathcal{A}(J)$
- $U(\gamma)\mathcal{A}(I)U(\gamma)^* = \mathcal{A}(\gamma(I))$
- Ω is fixed by $\text{Möb} = \mathrm{Aut}(\mathbb{D})$, cyclic for $\bigvee \mathcal{A}(I)$
- $I \cap J = \emptyset \Rightarrow \overbrace{\mathcal{A}(I) \subseteq \mathcal{A}(J)'}^{[\mathcal{A}(I), \mathcal{A}(J)] = 0}$
- if $\gamma|_I = id$, then $U(\gamma) \in \mathcal{A}(I')$
- the generator L_0 of rotation is positive

Consequences

- $\mathcal{H} = \bigoplus^{\ell^2}_{n \in \mathbb{Z}_+} V(n)$ $\quad V(n) = \mathrm{Ker}(L_0 - n)$

$V := \bigoplus^{alg}_{n \in \mathbb{Z}_+} V(n)$

We assume $\dim V(n) < \infty$; conjectured to always hold

- $\mathcal{A}(I)$ is the hyperfinite III_1 factor

- $\mathcal{A}(I') = \mathcal{A}(I)'$

$\quad = \{x \in B(\mathcal{H}) \mid [x, y] = 0$ for all $y \in \mathcal{A}(I)\}$

Vertex operator algebras

- Graded vector space $V = \bigoplus^{alg}_{n \in \mathbb{Z}_+} V(n)$
- state-field correspondence

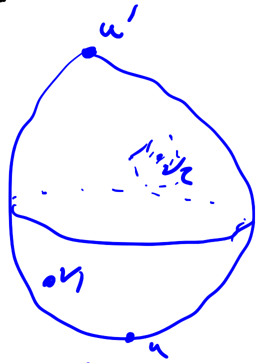
$$V \longrightarrow \mathrm{End}(V)[[z^{\pm 1}]]$$

$$v \longmapsto Y(v, z)$$

Key locality axiom:

$(z-w)^N [Y(v,z), Y(u,w)] = 0$

Capture correlation functions on the Riemann sphere:

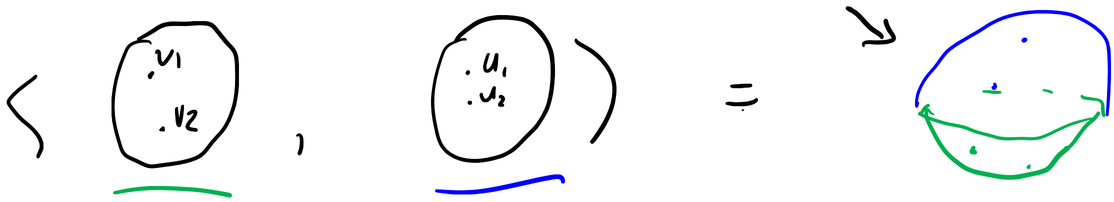


$\langle Y(v_2, z_2) Y(v_1, z_1) u, u' \rangle$

A unitary VOA has an inner product on V that is compatible with the fields.

$\rightsquigarrow Y(v_3, z_3) Y(v_2, z_2) Y(v_1, z_1) \Omega \in H := \bigoplus^{\ell^2}_{n \in \mathbb{Z}_+} V(n)$

$|z_3| > |z_2| > |z_1|$



How are these notions related?

Given a unitary VOA V, $f \in C^\infty(S^1)$

$$Y(v,f) := \int_{S^1} Y(v,z) f(z)\, dz$$

(Raymond-Tanimoto-T: $Y(v,f) : V \to \underline{\underline{H}}$)

Build a conformal net $A(I) = vN\big(\underline{Y(v,f)} \;\big|\; v \in V,\ \mathrm{supp}(f) \subseteq I\big)$

replace $Y(v,f)$ with "bounded measurable functions" of $Y(v,f)$ think: $e^{iY(v,f)}$

<u>Big Problem</u>: $A(I)$ and $A(J)$ may not commute when $I \cap J = \emptyset$.

Some prior work:

Carpi-Kawahigashi-Longo-Weiner '18: Built a framework for the smeared field approach to VOA $\to$ CN

not needed, RTT

VOA w/ "energy bounds" $\rightsquigarrow$ CN

A(I) and A(J) commute when $I \cap J = \emptyset$ "strong locality"

Theorem (Henriques–T)

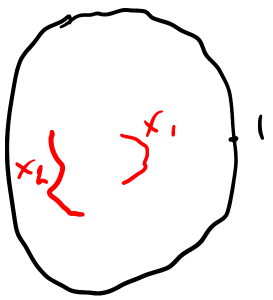
There is a bijection between conformal nets and "integrable" unitary VOAs.

Theorem (Henriques–T, Raymond–Tanimoto–T)

Every conformal net arises from the smeared field construction from a unitary VOA.

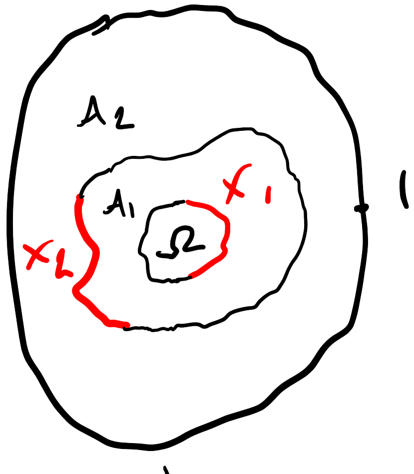
How does it work? CN $\Rightarrow$ VOA

Worm insertions



intervals, parametrized by intervals of S^1 I_1, I_2 labelled by $x_i \in A(I_i)$

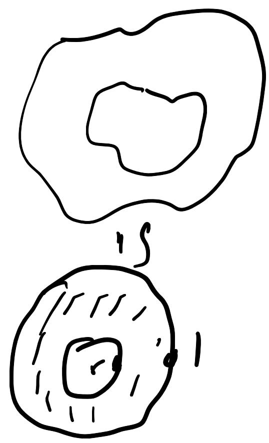
Assign to the picture a vector in the Hilbert space:



$$\| \quad A_2 x_2 A_1 x_1 \Omega$$

Key technical tool: Semigroup of annuli

$\mathbb{C}$-Annuli w/ parametrized ∂

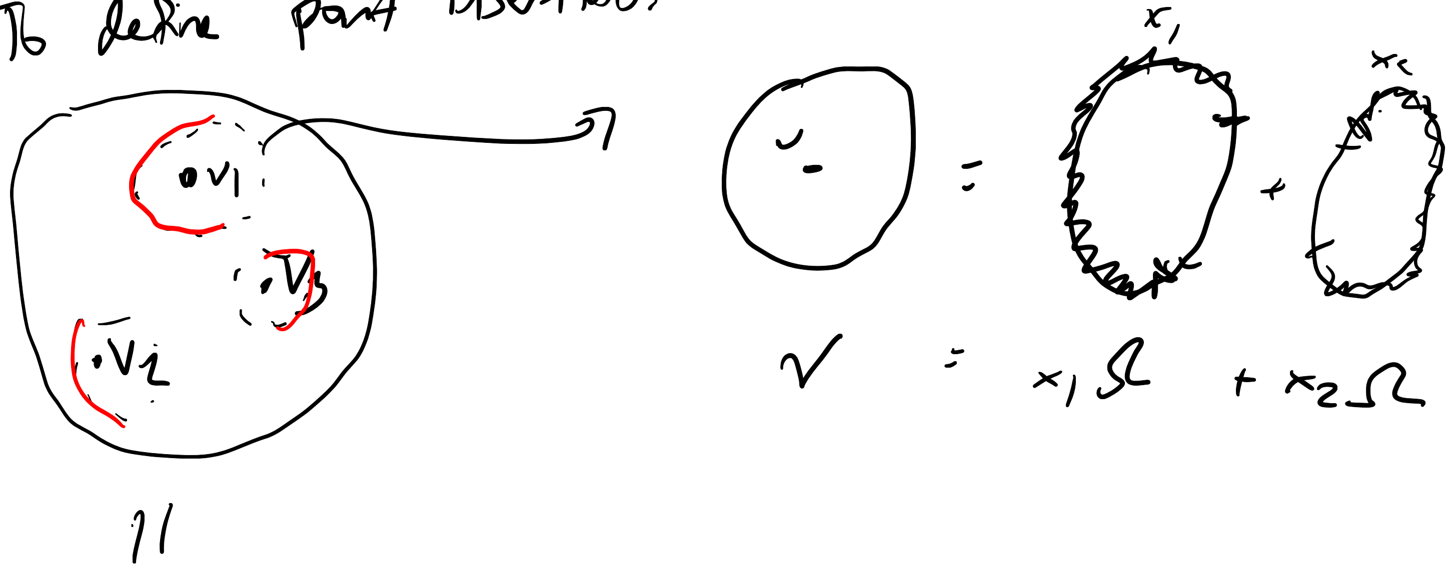


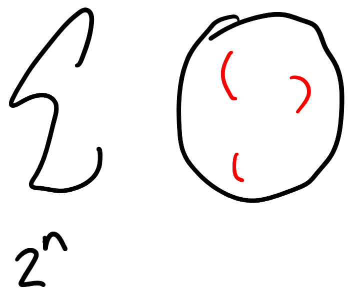
$\mapsto U(\gamma_{out}) r^{L_0} U(\gamma_{in})^*$

Lemma: Independent of choice of disks

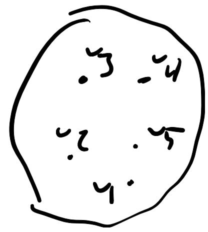
⇝ Requires "partially thin annuli"

To define point insertions:





This defines



Thm These point insertions define a VOA, which recovers the CN via smeared fields

Also "integrable VOA" constructions